An Accurate Model for Pull-in Voltage of Circular Diaphragm Capacitive Micromachined Ultrasonic Transducers (CMUT)

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Research Objective

Develop an accurate Analytical Model to calculate Pull-in Voltage of a CMUT device built with Circular diaphragm.

CMUT devices are a type of MEMS capacitive ultrasonic sensors characterized by small device dimensions.

The model takes into account

- Non-linear stretching of the diaphragm due to Large Deflection,
- Built-in Residual Stress of the membrane,
- Bending Stress, and
- the Fringing Field effect, and
- the non-linearity of the Electrostatic Field
Presentation Outline

- Motivation
- Basic Device Structure and Operation
- Limitations in Existing Models
- Solution Approach
- Model Development
- Model Validation
- Conclusions
CMUT devices has

- Wide Range of Applications
  - Biomedical Imaging
  - Automotive Collision Avoidance Radar System
  - Nondestructive Testing
  - Microphones

- Superior performance over traditional piezoelectric transducers
  - low temperature sensitivity
  - high signal sensitivity
  - wide bandwidth
  - very high level of integration (low cost)

Estimating Pull-in Voltage \( V_{\text{Pull-in}} \) and the Plate Travel Distance \( x_p \) before pull-in effect is required for the successful design of electrostatic actuators, switches, varactors, and sensors.
Basic Device Structure

Basic structure of a CMUT device with a circular diaphragm.
Operation Principle

Diaphragm deformation after subject to an external pressure.

At equilibrium, Electrostatic Attraction Force is balanced by the Elastic Restoring Force of the Diaphragm.
Diaphragm Collapse and Pull-in Voltage ($V_{\text{Pull-in}}$)

For $V < V_{\text{Pull-in}}$, \textit{Electrostatic Force} = \textit{Elastic Restoring Force}

For $V > V_{\text{Pull-in}}$, \textit{Electrostatic Force} > \textit{Elastic Restoring Force}

Diaphragm collapses under excessive electrostatic pressure

Collapsed Diaphragm
Limitations in Existing Models: Parallel Plate Approximation

- Existing models are based on parallel plate approximation
- Assumes piston-like motion of the diaphragm and predicts pull-in when center deflection reaches \(1/3\) of the airgap

\[
W_{0-PI} = 1/3d_0
\]  
Pull-in travel of the diaphragm

Deflection of Circular Diaphragm in Intellisuite
Limitations in Existing Models: Spring Hardening Effect

- Does not account for **spring hardening effect** due to non-linear stretching of the diaphragm under **large deflection**, more pronounced in **thin diaphragms** as in CMUTs

  \[ F = kx, \quad k \text{ is the spring constant} \]

  \[ w_{0-\text{PI}} \approx 0.5d_0 \]  \hspace{1cm} (2)

- Spring hardening causes the Pull-in Voltage to go up and center deflection can be as high as 50% of the airgap

- Does not take into account the effect of **Poisson ratio**
Limitations in Existing Models: Fringing Field Effect

- Existing model does take into account the effect due to Fringing Fields

- Fringing Field effect causes spring softening effect of the diaphragm, lowering the pull-in voltage

- The effect is more pronounced in sensors with small diaphragm, such as in CMUT devices

- Above simplifications results in an error as high as 20% when compared to experimental and FEA results
Solution Approach

- Develop \textit{Electrostatic Force Expression} that includes Fringing Field effect
- Linearize \textit{Electrostatic Force Expression} using Taylor Series Expansion
- Develop \textit{Load-Deflection Model} for large deflection
- Load-Deflection Model with \textit{Linearized Electrostatic Force}
- Apply \textit{Parallel Plate Approximation}
  \[ w_{0,pl} = d_0/3 \]
- Solve for \( V = V_{PI} \)
Model Development

Load-Deflection Model
Assumptions

- As the deflection of membrane is very small compared to its side-length, the forces on the membrane are assumed to always act perpendicular to the diaphragm surface.

- The deflection of the membrane is assumed to be a quasistatic process, i.e., the dynamic effects due to its motion (such as inertia force, damping force, etc) are not considered in this analysis.
Load-deflection model for circular membrane, subject to large deflection, due to an applied uniform external pressure $P$:

$$P = \left[ \frac{4\sigma_0 h}{a^2} + \frac{64D}{a^4} \right] w_o + \left[ \frac{128\alpha D}{h^2 a^4} \right] w_o^3$$ (3)

Combined Stiffness due to Bending and residual stress

Stiffness of the membrane due to the nonlinear stretching

$h$ $\longrightarrow$ Membrane Thickness

$D = \frac{\tilde{E}h^3}{12(1-\nu^2)}$ $\longrightarrow$ Flexural rigidity

$\tilde{E} = \frac{E}{1-\nu^2}$ $\longrightarrow$ Effective Young’s modulus

$\alpha = \frac{7505+4250\nu-2791\nu^2}{35280}$ $\longrightarrow$ Poisson ratio dependent empirical parameter

$\sigma_0$ $\longrightarrow$ Residual Stress

$E$ $\longrightarrow$ Young’s modulus

$a$ $\longrightarrow$ Membrane radius

$\nu$ $\longrightarrow$ Poisson ratio

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Model Development

Electrostatic Pressure
Electrostatic Force Profile

Electrostatic force profile in a parallel plate geometry:
Uniform but Non-Linear

Electrostatic force profile in a deformed clamped square diaphragm:
Non-Uniform and Non-Linear

No known solution for Load-Deflection Model exist with non-linear non-uniform electrostatic force
Solution Approach for Non-Linear Non-Uniform Force

Assume a **Piston-like motion** of the Diaphragm, CMUT can be approximated as **Parallel Plate Actuator**

- **Develop**
  - **Expression for Electrostatic Force**
    - Uniform But Non-Linear

- **Linearize**
  - **Electrostatic Force using Taylor Series Expansion**

- **Uniform Linear Electrostatic Force**
Capacitance: Undeflected Diaphragm

- Capacitance of undeflected diaphragm:

\[
C = C_0 \left(1 + C_{ff}\right) = \frac{a^2 \pi \varepsilon_r \varepsilon_0}{d_0} \left\{ 1 + \frac{2d_0}{\pi \varepsilon_r a} \left[ \ln \left( \frac{a}{2d_0} \right) + (1.41 \varepsilon_r + 1.77) + \frac{d_0}{a} (0.268 \varepsilon_r + 1.65) \right] \right\}
\]

\[C_0 = \varepsilon_0 \varepsilon_r \pi a^2/d_0\]  \quad \text{Parallel plate capacitance} \quad (5)

\[C_{ff} = \frac{2d_0}{\pi \varepsilon_r a} \left[ \ln \left( \frac{a}{2d_0} \right) + (1.41 \varepsilon_r + 1.77) + \frac{d_0}{a} (0.268 \varepsilon_r + 1.65) \right] \]  \quad \text{Fringing Field Factor} \quad (6)

- \(\varepsilon_0 = 8.854 \times 10^{-14}\)  \quad \text{Dielectric Permittivity of free space} \quad (7)

- \(\varepsilon_r = 1\) \text{ (for air)}  \quad \text{Relative Permittivity of the medium}

Capacitance: Deflected Diaphragm

- Capacitance of deflected diaphragm for any deflection $w$:

$$ C_{PP, Piston} \approx \frac{a^2 \pi \varepsilon_r \varepsilon_0}{d} \left\{ 1 + \frac{2d}{\pi \varepsilon_r a} \left[ \ln \left( \frac{a}{2d} \right) + (1.41 \varepsilon_r + 1.77) + \frac{d}{a} (0.268 \varepsilon_r + 1.65) \right] \right\} $$

where, $d = d_0 - w$, is the separation between the diaphragm and the backplate.

Assumption made

Piston-like motion of the diaphragm
(Parallel Plate Approximation)

- $\varepsilon_r = 1$ (for air), substituting in (8) the Capacitance expression becomes:

$$ C_{pp, piston} \approx \frac{a^2 \pi \varepsilon_0}{d_0 - w} \left\{ 1 + \frac{2(d_0 - w)}{\pi a} \left[ \ln \left( \frac{a}{2(d_0 - w)} \right) + 1.918 \frac{(d_0 - w)}{a} + 3.18 \right] \right\} $$
The developed *Electrostatic Force* after applying a bias voltage $V$

$$F_{\text{electrostatic}} = -\frac{d}{dz}\left(\frac{1}{2}C_{\text{pp-piston}}V^2\right) = \left[\frac{\pi a^2}{2(d_o-w)^2} + \frac{a}{d_o-w} - 1.918\right] \varepsilon_0 V^2$$

(10)

After Linearizing using Taylor series expansion method about the zero deflection point and rearranging the terms:

$$F_{\text{electrostatic}} = \varepsilon_0 \pi a^2 V^2 \left[\frac{1}{2d_o^2} + \frac{1}{\pi a d_o} + \frac{1.918}{\pi a^2}\right] + \varepsilon_0 \pi a^2 V^2 \left[\frac{1}{d_o^3} + \frac{1}{\pi a d_o^2}\right] w$$

(11)
Replacing \( w = w_0 \), the *Electrostatic Pressure* is given by,

\[
P_{\text{electrostatic}} = \frac{F_{\text{electrostatic}}}{A} = \varepsilon_0 V^2 \left[ \frac{1}{2d_0^2} + \frac{1}{\pi a d_0} \frac{1.918}{\pi a^2} \right] + \varepsilon_0 V^2 \left[ \frac{1}{d_0^3} + \frac{1}{\pi a d_0^2} \right] w_0 \tag{12}
\]

Load-deflection model due to electrostatic pressure

Yields uniform linear electrostatic pressure profile

Virtual planar diaphragm after linearizing the electrostatic force about the zero deflection point of the diaphragm center
Model Development

Pull-in Voltage
For Parallel Plate Actuator, diaphragm travel at pull-in

\[ w_{0-\text{pl}} = d_0/3 \]  \hspace{1cm} (1)

At pull-in equilibrium,

1. **Electrostatic Pressure** \( P_{\text{pl-electrostatic}} \) (from electrostatic pressure expression)
   [after substituting \( w_0 = d_0/3 \) in (12)]:

\[
P_{\text{pl-electrostatic}} = \varepsilon_0 V_{\text{PI}}^2 \left[ \frac{1}{2d_0^2} + \frac{1}{\pi ad_0} + \frac{1.918}{\pi a^2} \right] + \varepsilon_0 V_{\text{PI}}^2 \left[ \frac{1}{d_0^3} + \frac{1}{\pi ad_0^2} \right] \left( \frac{d_0}{3} \right)
\]  \hspace{1cm} (13)

2. **Elastic Restoring Pressure** \( P_{\text{pl-elastic}} \) (from load-deflection model)
   [after substituting \( w_0 = d_0/3 \) in (3)]

\[
P_{\text{pl-elastic}} = \left[ \frac{64D}{a^4} + \frac{4\sigma h}{a^2} \right] \left( \frac{d_0}{3} \right) + \frac{128\alpha D}{h^2 a^4} \left( \frac{d_0}{3} \right)^3
\]  \hspace{1cm} (14)
At *pull-in equilibrium*, the **electrostatic pressure** is just counterbalanced by the **elastic restoring pressure**

$$P_{\text{PI-electrostatic}} = P_{\text{PI-elastic}}$$ (15)

(13) and (14) can be solved simultaneously to yield the closed-form model for the pull-in voltage $V_{\text{PI}}$ for the circular diaphragm as:

$$V_{\text{PI}} = \sqrt{\frac{64D}{a^4} + \frac{4\sigma h}{a^2} \left(\frac{d_0}{3}\right) + \frac{128\alpha D}{h^2 a^4} \left(\frac{d_0}{3}\right)^3}$$

$$\varepsilon_0 \left[\frac{5}{6d_0^2} + \frac{4}{3\pi ad_0} + \frac{1.918}{\pi a^2}\right]$$ (16)
Model Validation
Finite Element Analysis Using IntelliSuite

Table I. Device Specifications Used in FEA Simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diaphragm Radius (a)</td>
<td>µm</td>
<td>250</td>
</tr>
<tr>
<td>Diaphragm Thickness (h)</td>
<td>µm</td>
<td>1-3</td>
</tr>
<tr>
<td>Airgap Thickness (d_0)</td>
<td>µm</td>
<td>2-3</td>
</tr>
<tr>
<td>Young’s Modulus (E)</td>
<td>GPa</td>
<td>169</td>
</tr>
<tr>
<td>Poisson Ratio (ν)</td>
<td>-</td>
<td>0.3</td>
</tr>
</tbody>
</table>

(Right) Screen capture of the diaphragm collapse after carrying out a 3-D electromechanical FEA using IntelliSuite. \((h = 3 \text{ µm}, d_0 = 3 \text{ µm}, \sigma_0 = 100 \text{ MPa})\).
### Table II. Pull-in Voltage Comparison

Diaphragm thickness, \( h =1 \mu \text{m} \), Airgap thickness, \( d_0 =2 \mu \text{m} \)

<table>
<thead>
<tr>
<th>Residual stress (MPa)</th>
<th>(V)</th>
<th>( \Delta % )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>New Model</td>
<td>Ref. [14]</td>
</tr>
<tr>
<td>0</td>
<td>11.03</td>
<td>9.89</td>
</tr>
<tr>
<td>100</td>
<td>49.24</td>
<td>50.48</td>
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<tr>
<td>250</td>
<td>76.69</td>
<td>77.88</td>
</tr>
<tr>
<td>350</td>
<td>90.46</td>
<td>91.56</td>
</tr>
</tbody>
</table>

Diaphragm thickness, \( h =1 \mu \text{m} \), Airgap thickness, \( d_0 =3 \mu \text{m} \)

<table>
<thead>
<tr>
<th>Residual stress (MPa)</th>
<th>(V)</th>
<th>( \Delta % )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>New Model</td>
<td>Ref. [14]</td>
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<tr>
<td>0</td>
<td>22.38</td>
<td>18.17</td>
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<tr>
<td>100</td>
<td>90.88</td>
<td>92.83</td>
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<tr>
<td>250</td>
<td>141.06</td>
<td>143.07</td>
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<tr>
<td>350</td>
<td>166.3</td>
<td>168.2</td>
</tr>
</tbody>
</table>

# Table III. Pull-in Voltage Comparison

Diaphragm thickness, $h = 2 \, \mu m$, Airgap thickness, $d_0 = 3 \, \mu m$

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>55.03</td>
<td>51.39</td>
<td>57.6</td>
<td>4.45</td>
<td>10.77</td>
</tr>
<tr>
<td>100</td>
<td>136.18</td>
<td>141.18</td>
<td>138.4</td>
<td>1.6</td>
<td>2.0</td>
</tr>
<tr>
<td>250</td>
<td>204.5</td>
<td>211.24</td>
<td>205.6</td>
<td>0.53</td>
<td>2.74</td>
</tr>
<tr>
<td>350</td>
<td>239.45</td>
<td>246.53</td>
<td>239.8</td>
<td>0.15</td>
<td>2.8</td>
</tr>
</tbody>
</table>

Diaphragm thickness, $h = 3 \, \mu m$, Airgap thickness, $d_0 = 3 \, \mu m$

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>98.03</td>
<td>94.42</td>
<td>97.7</td>
<td>0.33</td>
<td>3.36</td>
</tr>
<tr>
<td>100</td>
<td>181.34</td>
<td>188.14</td>
<td>184.2</td>
<td>1.55</td>
<td>2.14</td>
</tr>
<tr>
<td>250</td>
<td>260.38</td>
<td>271.20</td>
<td>263.4</td>
<td>1.14</td>
<td>2.96</td>
</tr>
<tr>
<td>350</td>
<td>301.78</td>
<td>313.77</td>
<td>304.4</td>
<td>0.85</td>
<td>3.07</td>
</tr>
</tbody>
</table>

Model Validation

Pull-in Voltage vs Diaphragm Radius ($\sigma_0 > 0$)

(Diaphragm thickness, $h = 3 \, \mu m$, Airgap thickness, $d_0 = 3 \, \mu m$, Residual stress $\sigma_0 = 250 \, MPa$)

Both the models merge at large diaphragm radius
Model Validation

**Pull-in Voltage vs Diaphragm Radius (σ₀=0)**

(Diaphragm thickness, \( h = 3 \mu\text{m} \), Airgap thickness, \( d_0 = 3 \mu\text{m} \), Residual stress \( \sigma_0 = 0 \))

Pull–in Voltage Comparison for Circular Diaphragms (σ=0)

- Ref. [14]
- New Analytical Model

Both the models merge at large diaphragm radius
The new analytical method can provide very good approximation of pull-in voltage for the following limited cases:

- Both the electrodes are required to be parallel prior to any electrostatic actuation.

- The gap between the clamped diaphragm and the backplate should be small enough so that the Taylor series expansion about the zero deflection point doesn’t introduce any significant error.

- The lateral dimensions of the diaphragm are required to be very large compared to the diaphragm’s thickness and the airgap.
Conclusions

- A highly accurate closed-form model for the pull-in voltage of a clamped circular diaphragm CMUT device is presented.

- The model is simple, easy to use and fast, and takes into account both the non-linear stretching due to large deflection and the non-linearity of electrostatic field.

- Excellent agreement with 3-D electromechanical FEA using IntelliSuite with a maximum deviation of 1.6% for diaphragms with residual stress.

- Can also be easily extended to a Clamped-edge Square Membrane.

- Besides CMUTs, the Model will also be useful in ensuring safe and efficient operation of
  - MEMS capacitive type pressure sensors
  - MEMS based microphones
  - Touch mode pressure sensors
  - Other application areas where electrostatically actuated circular diaphragms are used.
Thanks for your Patience