Computationally-Efficient Methods for Blind Adaptive Equalization

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Outline

1. Introduction
2. Computationally-Efficient Methods
3. Proposed Selective Update Method
4. Simulation Results
5. Conclusions
6. References
Introduction

- Blind adaptive equalization is used in systems where the transmission of a training sequence is impractical.
- Common blind algorithms include the reduced constellation algorithm (RCA), the constant modulus algorithm (CMA), and the multimodulus algorithm (MMA).
- Equalization can consume in excess of 80% of the total arithmetic computations needed to demodulate a transmitted symbol sequence into binary words, which has resulted in a number of computationally-efficient methods.
- We present a survey of efficient methods for blind equalization and propose a new method that selectively updates the equalizer taps based on the equalizer output radius for QAM signal constellations.
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Computationally-Efficient Methods

- Adaptive filtering consists of two operations: convolution of the received symbol sequence with the tap coefficients and updating the tap coefficients.

- For an adaptive FIR filter of length $M$, each of the previous operations require $4M$ multiplications for a total of $8M$ multiplications when the received signal is complex.

- One method to improve computational efficiency is to simplify or reduce the amount of multiplications.

- Our focus is the reduction of multiplications in the equalizer tap update and we consider the signed-error, dithered signed-error, quantized-error, block, and update decimation methods.
Adaptive FIR Filter Structure

Figure 1: Adaptive FIR filter for real input samples
Signed-Error Method

- Only the sign of the respective error signal is retained
- When coupled with a power-of-two stepsize, a multiply-free fixed-point equalizer tap update can be realized reducing the total multiplications by a factor of two
- The general signed-error tap update algorithm is:

\[ f_{n+1} = f_n + \mu \cdot \text{csgn}(e_n) r_n^* \]

- Signed-error algorithms are straightforward to implement and have been proposed for RCA and CMA, and can be extended to MMA
Dithered Signed-Error Method

- The convergence of signed-error CMA is not robust and is known to diverge.
- This can be overcome by the application of a controlled noise or dither signal, which improves robustness.
- The general dithered signed-error tap update algorithm is:

\[
f_{n+1} = f_n + \mu \cdot \alpha \text{sgn}(e_n + \alpha d_n) r_n^*
\]

- Where \( \alpha \) is a positive constant and \( d_n \) is an independent identically distributed (i.i.d.) dithering process uniformly distributed over (-1,1].
Quantized-Error Method

- The error signal of the respective algorithm is quantized using a nonlinear power-of-two quantizer.
- When coupled with a power-of-two stepsize the equalizer tap update becomes shift and add operations.
- The general quantized-error tap update algorithm is:
  \[ f_{n+1} = f_n + \mu \cdot Q\{e_n\} r^*_n \]
  - Where
    \[ Q\{x\} = \begin{cases} 
      \text{sgn}(x), & |x| \geq 1 \\
      2^{[\log_2|x|]} \text{sgn}(x), & 2^{-B+2} \leq |x| < 1 \\
      \tau \text{sgn}(x), & |x| < 2^{-B+2} 
    \end{cases} \]
  - And \( \tau \) is set to either 0 or \( 2^{-B+1} \) and \( B \) is the data word length.
Block Method

- A block of equalizer input samples and instantaneous error samples are used to update the tap coefficients once every $L$ input samples, where $L$ is the block length.

- The general block tap update algorithm is:

$$f_{(n+1)L} = f_{nL} + \mu \cdot \sum_{k=0}^{L-1} e_{nL+k} \cdot r_{nL+k}^*$$

- Estimates the gradient over $L$ iterations, which allows a larger stepsize to be applied since the variance of a block of gradient updates is less than that for individual updates.

- Can be implemented in frequency domain to increase rate of convergence.

- Have been proposed for CMA and can be extended to MMA.
Update Decimation Method

- The equalizer taps are updated once every $k$ iterations, where $k$ is a positive integer greater than one.
- It is expected that update-decimated algorithms would obtain similar steady-state mean-squared error (MSE) with $1/k$ times the computations, while taking $k$ times the time-to-convergence.
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Selective Update Method

- The square decision region of an estimated symbol point in a QAM constellation is divided in two by a circular boundary, $C_b$, which corresponds to radius $R_b$

- Equalizer taps are updated only if $R_n > R_b$, where $R_n$ is the distance from the estimated symbol to the equalizer output defined as:

  \[ R_n = |\hat{s}_n - Y_n| \]

- The general selective update tap update algorithm is:

  \[ f_{n+1} = f_n + \mu \cdot \psi(Y_n, R_n) r_n^* \]

- where

  \[ \psi(Y_n, R_n) = \begin{cases} 
  e_n, & R_n > R_b \\
  0, & R_n \leq R_b 
\end{cases} \]
Selective Update Method

Figure 2: Decision regions for symbol estimates in 16-QAM (left) and decision regions for the selective update method (right).
Selective Update Method

- The outer region corresponds to adaptation phases with high MSE, while the inner region corresponds to adaptation phases with low MSE.
- Initially, the MSE will be high and the outer region will be selected most of the time, allowing the transient response of the base algorithm to remain unchanged.
- In slow time-varying channels, once the MSE has been reduced, the inner region will be selected most of the time, which will result in a drastic reduction of tap updates.
- If the channel experiences sudden changes, the MSE will increase and the process will repeat.
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Simulation Parameters

- Discussed and proposed methods applied to CMA & MMA
- Simulations are in a 35dB SNR environment for 16-QAM using SPIB microwave channels (#1,2,4-6,8-10), with T/2-spaced FIR equalizers (16-tap, double 0.5 center spike)
- Applied stepsize of $2^{-10}$ (except DSE-CMA which used $2^{-11}$ to avoid divergence), block length $L=20$, $\alpha=0.65$
- $R_b$ for selective update method was chosen using an ad hoc approach and ranged between $d/8$ and $d/12$, where $d$ is the distance between symbol points
- MSE calculated as instantaneous squared error over the slicer for 100-1000 iterations
- Graphical results shown for SPIB microwave channel #2
Simulation Results for CMA-Based Algorithms

Figure 3: CMA simulation results.
Simulation Results for MMA-Based Algorithms

Figure 3: MMA simulation results.
Simulation Results

- Quantitative results have been averaged over all channels.
- In the table to follow, the MSE corresponds to the steady-state MSE, M is the misadjustment, and TTC is the time-to-convergence which was taken as the number of samples required to reach 90% of the steady-state MSE.
- Misadjustment is the ratio of excess MSE (EMSE) to the minimum theoretical MSE (MMSE), where EMSE is the difference between the steady-state MSE and the MMSE.
Quantitative Simulation Results

Table 1: Quantitative Simulation Results.

<table>
<thead>
<tr>
<th>Method</th>
<th>Performance Measures</th>
<th>Tap Update Percentage</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>MSE</td>
<td>M</td>
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<td>CMA</td>
<td>-23.8834</td>
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<td>DSE-CMA</td>
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<td>QE-CMA</td>
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<tr>
<td>SU-MMA</td>
<td>-27.5051</td>
<td>0.0888</td>
</tr>
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</table>

Proposed algorithms have the lowest misadjustment and same rate of convergence as original algorithms
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Conclusions

- Simulations have confirmed that on average, the proposed selective update method achieves similar transient behavior and lower steady-state MSE and misadjustment than the original algorithm.
- After convergence, the percentage of tap updates for the selective update method is considerably reduced (<15%).
- Performance gains obtained using the selective update method serve to validate this technique as being computationally-efficient as well as an effective method for blind equalization.
Thank You!
Questions or Comments?
References


References


References


